

## B.A. (Prog.) with Mathematics as Major First Semester

### Category II

#### **DISCIPLINE SPECIFIC CORE COURSE – 1: ELEMENTS OF DISCRETE MATHEMATICS**

#### **CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Discrete Mathematics	4	3	1	0	Class XII pass with Mathematics	Nil

#### **Learning Objectives**

Students are introducing to:

- Order (or partial order) and related properties.
- Notion of a lattice which is also a step towards abstract algebra.
- Concept of Boolean algebra and its applications to minimizing a Boolean polynomial and switching circuits, which has further applications in computer science.

#### **Learning outcomes**

This course will enable the students to:

- Understand the basic concepts of sets, relations, functions, and induction.
- Understand mathematical logic and logical operations to various fields.
- Understand the notion of order and maps between partially ordered sets.
- Minimize a Boolean polynomial and apply Boolean algebra techniques to decode switching circuits.

#### **SYLLABUS OF DSC - 1**

##### **Theory**

##### **Unit – 1**

**(24 hours)**

##### **Sets, Relations and Functions**

Sets, Propositions and logical operations, Conditional statements, Mathematical induction, Relations and equivalence relation, Equivalence classes, Partial order relation, Partially ordered set, Hasse diagrams, Chain, Maximal and minimal elements, least and greatest elements, Least upper bound, Greatest lower bound, Zorn's lemma, Functions and bijective functions, Functions between POSETS, Order isomorphism.

##### **Unit – 2**

**(16 hours)**

##### **Lattices**

Lattice as a POSET, Lattice as an algebra and their equivalence, Bounded lattices, Sublattices, Interval in a lattice, Products and homomorphism of lattices, Isomorphism of lattices; Distributive, Complemented, Partition and pentagonal lattices.

### Unit – 3

(20 hours)

#### Boolean Algebra and Switching Circuits

Boolean algebra, De Morgan's laws, Boolean expressions, Truth tables, Logic diagrams, Boolean functions, Disjunctive normal forms (as join of meets), Minimal forms of Boolean polynomials, Quine Mc-Cluskey method, Karnaugh maps, Switching circuits, Applications of switching circuits.

**Practical component (if any) – NIL**

#### Essential Readings

- Rudolf Lidl, & Gunter Pilz (2004). Applied Abstract Algebra (2nd ed.). Undergraduate text in Mathematics, Springer (SIE), Indian Reprint.
- Bernard Kolman, Robert C. Busby, & Sharon Cutler Ross (2009). Discrete Mathematical Structures (6th ed.). Pearson education Inc., Indian reprint.

#### Suggestive Reading

- Rosen, Kenneth H. (2017). Discrete Mathematics and its applications with combinatorics and Graph Theory (7th ed.). McGraw Hill Education.

**Note:** Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

## DISCIPLINE SPECIFIC CORE COURSE – 2: TOPICS IN CALCULUS

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Topics in Calculus	4	3	1	0	Class XII pass with Mathematics	Nil

#### Learning Objectives

The primary objective of this course is to:

- Introduce the basic tools of calculus which are helpful in understanding their applications in many real-world problems.
- Understand/create various mathematical models in everyday life.

#### Learning Outcomes

This course will enable the students to:

- Understand continuity and differentiability in terms of limits and graphs of certain functions.
- Describe asymptotic behaviour in terms of limits involving infinity.
- Use of derivatives to explore the behaviour of a given function locating and classify its

extrema and graphing the function.

- Apply the concepts of asymptotes, and inflexion points in tracing of cartesian curves.
- Compute the reduction formulae of standard transcendental functions with applications.

## SYLLABUS OF DSC - 2

### Theory

#### Unit – 1 (20 hours)

##### Limits, Continuity and Differentiability

Limit of a function,  $\varepsilon$ - $\delta$  definition of a limit, Infinite limits, Continuity and types of discontinuities; Differentiability of a function, Successive differentiation: Calculation of the  $n$ th derivatives, Leibnitz theorem; Partial differentiation, Euler's theorem on homogeneous functions.

#### Unit – 2 (20 hours)

##### Mean Value Theorems and its Applications

Rolle's theorem, Mean value theorems and applications to monotonic functions and inequalities; Taylor's theorem, Taylor's series, Maclaurin's series expansions of  $e^x$ ,  $\sin x$ ,  $\cos x$ ,  $\log(1+x)$  and  $(1+x)^m$ ; Indeterminate forms.

#### Unit – 3 (20 hours)

##### Tracing of Curves and Reduction Formulae

Asymptotes (parallel to axes and oblique), Concavity and inflexion points, Singular points, Tangents at the origin and nature of singular points, Curve tracing (cartesian and polar equations). Reduction formulae for  $\int \sin^n x dx$ ,  $\int \cos^n x dx$ , and  $\int \sin^m x \cos^n x dx$  and their applications.

#### Practical component (if any) – NIL

#### Essential Readings

- Prasad, Gorakh (2016). Differential Calculus (19th ed.). Pothishala Pvt. Ltd. Allahabad.
- Prasad, Gorakh (2015). Integral Calculus. Pothishala Pvt. Ltd. Allahabad.

#### Suggestive Readings

- Apostol, T. M. (2007). Calculus: One-Variable Calculus with An Introduction to Linear Algebra (2nd ed.). Vol. 1. Wiley India Pvt. Ltd.
- Ross, Kenneth. A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian reprint.

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B.A/ B.Sc. (Prog.) with Mathematics as Non-Major First Semester

Category III

**DISCIPLINE SPECIFIC CORE COURSE: TOPICS IN CALCULUS**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Topics in Calculus	4	3	1	0	Class XII pass with Mathematics	Nil

**Learning Objectives**

The primary objective of this course is to:

- Introduce the basic tools of calculus which are helpful in understanding their applications in many real-world problems.
- Understand/create various mathematical models in everyday life.

**Learning outcomes**

This course will enable the students to:

- Understand continuity and differentiability in terms of limits and graphs of certain functions.
- Describe asymptotic behaviour in terms of limits involving infinity.
- Use of derivatives to explore the behaviour of a given function locating and classify its extrema and graphing the function.
- Apply the concepts of asymptotes, and inflexion points in tracing of cartesian curves.
- Compute the reduction formulae of standard transcendental functions with applications.

**SYLLABUS OF DSC**

**Theory**

**Unit – 1 (20 hours)**

**Limits, Continuity and Differentiability**

Limit of a function,  $\varepsilon - \delta$  definition of a limit, Infinite limits, Continuity and types of discontinuities; Differentiability of a function, Successive differentiation: Calculation of the nth derivatives, Leibnitz theorem; Partial differentiation, Euler's theorem on homogeneous functions.

**Unit – 2 (20 hours)**

### Mean Value Theorems and its Applications

Rolle's theorem, Mean value theorems and applications to monotonic functions and inequalities; Taylor's theorem, Taylor's series, Maclaurin's series expansions of  $e^x$ ,  $\sin x$ ,  $\cos x$ ,  $\log(1+x)$  and  $(1+x)^m$ ; Indeterminate forms.

### Unit – 3

(20 hours)

#### Tracing of Curves and Reduction Formulae

Asymptotes (parallel to axes and oblique), Concavity and inflexion points, Singular points, Tangents at the origin and nature of singular points, Curve tracing (cartesian and polar equations). Reduction formulae for  $\int \sin^n x dx$ ,  $\int \cos^n x dx$ , and  $\int \sin^m x \cos^n x dx$  and their applications.

**Practical component (if any) – NIL**

#### Essential Readings

- Prasad, Gorakh (2016). Differential Calculus (19th ed.). Pothishala Pvt. Ltd. Allahabad.
- Prasad, Gorakh (2015). Integral Calculus. Pothishala Pvt. Ltd. Allahabad.

#### Suggestive Readings

- Apostol, T. M. (2007). Calculus: One-Variable Calculus with An Introduction to Linear Algebra (2nd ed.). Vol. 1. Wiley India Pvt. Ltd.
- Ross, Kenneth. A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian reprint.

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**BA. (Prog.) with Mathematics as Major** Second Semester

**Category II**

**DISCIPLINE SPECIFIC CORE COURSE (DSC-2): ANALYTIC GEOMETRY**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Analytic Geometry	4	3	1	0	Class XII pass with Mathematics	NIL

**Learning Objectives:** The course aims at:

- Identifying and sketching curves, studying three dimensional objects, their geometric properties and applications.
- Use of vector approach to three-dimensional geometry makes the study simple and elegant.

**Learning Outcomes:** This course will enable the students to:

- Learn concepts in two-dimensional geometry.
- Identify and sketch conics namely, ellipse, parabola and hyperbola.
- Learn about three-dimensional objects such as straight lines and planes using vectors, spheres, cones and cylinders.

**SYLLABUS OF DSC-2**

**UNIT – I: Conic Sections** **(15 hours)**

Techniques for sketching parabola, ellipse and hyperbola; Reflection properties of parabola, ellipse, hyperbola, and their applications to signals; Classification of quadratic equation representing lines, parabola, ellipse and hyperbola; Rotation of axes; Second degree equations.

**UNIT – II: Vectors, Lines and Planes** **(18 hours)**

Rectangular coordinates in 3-dimensional space, vectors viewed geometrically, vectors in coordinate systems and vectors determined by length and angle; Dot product; Projections; Cross product, scalar triple product, vector triple product and their geometrical properties; Parametric equations of lines, direction cosines and direction ratios of a line, vector and symmetric equations of lines, angle between two lines; Planes in 3-dimensional space, coplanarity of two lines, angle between two planes, distance of a point from a plane, angle between a line and a plane, distance between parallel planes; Shortest distance between two skew lines.

**UNIT – III: Sphere, Cone and Cylinder****(12 hours)**

Equation of a sphere, plane section of sphere, tangents and tangent plane to a sphere; Equation of a cone, enveloping cone of a sphere, Reciprocal cones and right circular cone; Equation of a cylinder, enveloping cylinder and right circular cylinder.

**Essential Readings**

1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pte. Ltd. Indian reprint (2016) by Wiley India Pvt. Ltd. Delhi.
2. Narayan, Shanti & Mittal, P. K. (2007). *Analytical Solid Geometry*. S. Chand & Company Pvt Ltd. India.

**Suggestive Readings**

- Bell, Robert J.T. (1972). *An Elementary Treatise on Coordinate Geometry of Three Dimensions*. Macmillan & Co. Ltd. London.
- George B. Thomas, Jr., & Ross L. Finney (2012). *Calculus and Analytic Geometry* (9th ed.). Pearson Indian Education Services Pvt Ltd. India.

**DISCIPLINE SPECIFIC CORE COURSE – 2 (Discipline A-2): Elementary Linear Algebra**
**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elementary Linear Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

**Learning Objectives:** The objective of the course is:

- To introduce the concept of vectors in  $R^n$ .
- Understanding the nature of solution of system of linear equations.
- To view the  $m \times n$  matrices as a linear function from  $R^n$  to  $R^m$  and vice versa.
- To introduce the concepts of linear independence and dependence, rank and linear transformations has been explained through matrices.

**Learning Outcomes:** This course will enable the students to:

- Visualize the space  $R^n$  in terms of vectors and the interrelation of vectors with matrices.
- Familiarize with concepts of bases, dimension and minimal spanning sets in vector spaces.
- Learn about linear transformation and its corresponding matrix.

## SYLLABUS OF DSC-2

### UNIT – I: Euclidean Space $R^n$ and Matrices (18 hours)

Fundamental operations with vectors in Euclidean space  $R^n$ , Linear combinations of vectors, Dot product and their properties, Cauchy-Schwarz inequality, Triangle inequality, Solving system of linear equations using Gaussian elimination, Application: Curve Fitting, Gauss-Jordan row reduction, Reduced row echelon form, Application: Solving several systems simultaneously, Equivalent systems, Rank and row space of a matrix, Eigenvalues, Eigenvectors, Eigenspace, Diagonalization, Characteristic polynomial of a matrix.

### UNIT – II: Introduction to Vector Spaces (12 hours)

Definition, Examples and some elementary properties of vector spaces, Subspaces, Span, Linear independence and linear dependence of vectors, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets.

### UNIT – II: Linear Transformations (15 hours)

Linear transformations: Definition, Examples and elementary properties, The matrix of a linear transformation, Kernel and range of a linear transformation, The dimension theorem, one-to-one and onto linear transformations, Invertible linear transformations, Isomorphic vector spaces.

#### Essential Reading

1. Andrilli, S., & Hecker, D. (2016). *Elementary Linear Algebra* (5th ed.). Elsevier India.

#### Suggestive Readings

- Lay, David C., Lay, Steven R., & McDonald, Judi J. (2016). *Linear Algebra and its Applications* (5th ed.). Pearson Education.
- Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.

## B.Sc. (Prog.)/ BA (Prog.) with Mathematics as Non-Major **Second Semester** *Category-III*

### DISCIPLINE SPECIFIC CORE COURSE – 2 (Discipline A-2): Elementary Linear Algebra

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elementary Linear Algebra	4	3	1	0	Class XII pass with Mathematics	NIL



**Learning Objectives:** The objective of the course is:

- To introduce the concept of vectors in  $R^n$ .
- Understand the nature of solution of system of linear equations.
- To view the  $m \times n$  matrices as a linear function from  $R^n$  to  $R^m$  and vice versa.
- To introduce the concepts of linear independence and dependence, rank and linear transformations has been explained through matrices.

**Learning Outcomes:** This course will enable the students to:

- Visualize the space  $R^n$  in terms of vectors and the interrelation of vectors with matrices.
- Familiarize with concepts of bases, dimension and minimal spanning sets in vector spaces.
- Learn about linear transformation and its corresponding matrix.

## SYLLABUS OF DSC-2

### UNIT – I: Euclidean Space $R^n$ and Matrices (18 hours)

Fundamental operations with vectors in Euclidean space  $R^n$ , Linear combinations of vectors, Dot product and their properties, Cauchy-Schwarz inequality, Triangle inequality, Solving system of linear equations using Gaussian elimination, Application: Curve Fitting, Gauss-Jordan row reduction, Reduced row echelon form, Application: Solving several systems simultaneously, Equivalent systems, Rank and row space of a matrix, Eigenvalues, Eigenvectors, Eigenspace, Diagonalization, Characteristic polynomial of a matrix.

### UNIT – II: Introduction to Vector Spaces (12 hours)

Definition, Examples and some elementary properties of vector spaces, Subspaces, Span, Linear independence and linear dependence of vectors, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets.

### UNIT – III: Linear Transformations (15 hours)

Linear transformations: Definition, Examples and elementary properties, The matrix of a linear transformation, Kernel and range of a linear transformation, The dimension theorem, one-to-one and onto linear transformations, Invertible linear transformations, Isomorphic vector spaces.

#### Essential Reading

1. Andrilli, S., & Hecker, D. (2016). *Elementary Linear Algebra* (5th ed.). Elsevier India.

#### Suggestive Readings

- Lay, David C., Lay, Steven R., & McDonald, Judi J. (2016). *Linear Algebra and its Applications* (5th ed.). Pearson Education.
- Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.

**Category-II**

**DISCIPLINE SPECIFIC CORE COURSE – 3:  
THEORY OF EQUATIONS AND SYMMETRIES**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Theory of Equations and Symmetries	4	3	1	0	Class X pass with Mathematics	Nil

**Learning Objectives**

The goal of this paper is to acquaint students with certain ideas about:

- Integral roots, rational roots, an upper bound on number of positive or negative roots of a polynomial.
- Finding roots of cubic and quartic equations in special cases using elementary symmetric functions.
- Using Cardon's and Descartes' methods, respectively.

**Learning Outcomes**

After completion of this paper, the students will be able to:

- Understand the nature of the roots of polynomial equations and their symmetries.
- Solve cubic and quartic polynomial equations with special condition on roots and in general.
- Find symmetric functions in terms of the elementary symmetric polynomials.

**SYLLABUS OF DSC-3**

**Unit – 1 (18 hours)**

**Polynomial Equations and Properties**

General properties of polynomials and equations; Fundamental theorem of algebra and its consequences; Theorems on imaginary, integral and rational roots; Descartes' rule of signs for positive and negative roots; Relations between the roots and coefficients of equations, Applications to solution of equations when an additional relation among the roots is given; De Moivre's theorem for rational indices, the  $n$ th roots of unity and symmetries of the solutions.

**Unit – 2 (12 hours)**

**Cubic and Biquadratic (Quartic) Equations**

Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Cardon’s method of solving cubic and Descartes’ method of solving biquadratic equations.

### Unit – 3

(15 hours)

#### Symmetric Functions

Elementary symmetric functions and symmetric functions of the roots of an equation; Newton’s theorem on sums of the like powers of the roots; Computation of symmetric

functions such as  $\sum \alpha^2 \beta$ ,  $\sum \alpha^2 \beta^2$ ,  $\sum \alpha^2 \beta \gamma$ ,  $\sum \frac{1}{\alpha^2 \beta \gamma}$ ,  $\sum \alpha^{-3}$ ,  $\sum (\beta + \gamma - \alpha)^2$ ,  $\sum \frac{\alpha^2 + \beta \gamma}{\beta + \gamma}$ , ... of polynomial equations; Transformation of equations by symmetric functions and in general.

#### Essential Readings

1. Burnside, W.S., & Panton, A.W. (1979). The Theory of Equations (11th ed.). Vol. 1. Dover Publications, Inc. (4th Indian reprint. S. Chand & Co. New Delhi).
2. Dickson, Leonard Eugene (2009). First Course in the Theory of Equations. John Wiley & Sons, Inc. The Project Gutenberg eBook: <http://www.gutenberg.org/ebooks/29785>

#### Suggestive Readings

- Prasad, Chandrika (2017). Text Book of Algebra and Theory of Equations. Pothishala Pvt Ltd.

**Note:** Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

### DISCIPLINE SPECIFIC CORE COURSE – A-3: DIFFERENTIAL EQUATIONS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Differential Equations	4	3	1	0	Class XII pass with Mathematics	Nil

#### Learning Objectives

The primary objective of this course is to introduce:

- Ordinary and partial differential equations.
- Basic theory of higher order linear differential equations, Wronskian and its properties.
- Various techniques to find the solutions of above differential equations which provide a basis to model complex real-world situations.

#### Learning Outcomes

This course will enable the students to:

- Solve the exact, linear, Bernoulli equations, find orthogonal trajectories and solve rate problems.

- Apply the method of undetermined coefficients and variation of parameters to solve linear differential equations.
- Solve Cauchy-Euler equations and system of linear differential equations.
- Formulate and solve various types of first and second order partial differential equations.

## SYLLABUS OF DISCIPLINE A-3

### Unit – 1 (15 hours)

#### Ordinary Differential Equations

First order ordinary differential equations: Basic concepts and ideas, First order Exact differential equations, Integrating factors and rules to find integrating factors, Linear equations and Bernoulli equations, Initial value problems, Applications of first order differential equations: Orthogonal trajectories and Rate problems; Basic theory of higher order linear differential equations, Wronskian and its properties.

### Unit – 2 (12 hours)

#### Explicit Methods of Solving Higher-Order Linear Differential Equations

Linear homogeneous equations with constant coefficients, Linear non-homogeneous equations, Method of undetermined coefficients, Method of variation of parameters, Two-point boundary value problems, Cauchy-Euler equations, System of linear differential equations.

### Unit – 3 (18 hours)

#### First and Second Order Partial Differential Equations

Classification and Construction of first-order partial differential equations, Method of characteristics and general solutions of first-order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations; Classification and reduction to canonical forms of second-order linear partial differential equations and their general solutions.

#### Essential Readings

1. Myint-U, Tyn and Debnath, Lokenath (2007). Linear Partial Differential Equations for Scientist and Engineers (4th ed.). Birkhäuser. Indian Reprint.
2. Ross, Shepley L. (1984). Differential Equations (3rd ed.). John Wiley & Sons.

#### Suggestive Readings

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson Education.
- Kreyszig, Erwin. (2011). Advanced Engineering Mathematics (10th ed.). Wiley India.
- Sneddon I. N. (2006). Elements of Partial Differential Equations. Dover Publications.

**Note:** Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

**Category-III**

**DISCIPLINE SPECIFIC CORE COURSE – A-3:  
DIFFERENTIAL EQUATIONS**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Differential Equations	4	3	1	0	Class XII pass with Mathematics	Nil

**Learning Objectives**

The primary objective of this course is to introduce:

- Ordinary and partial differential equations.
- Basic theory of higher order linear differential equations, Wronskian and its properties.
- Various techniques to find the solutions of above differential equations which provide a basis to model complex real-world situations.

**Learning Outcomes**

This course will enable the students to:

- Solve the exact, linear, Bernoulli equations, find orthogonal trajectories and solve rate problems.
- Apply the method of undetermined coefficients and variation of parameters to solve linear differential equations.
- Solve Cauchy-Euler equations and System of linear differential equations.
- Formulate and solve various types of first and second order partial differential equations.

**SYLLABUS of Discipline A-3**

**Unit – 1 (15 hours)**

**Ordinary Differential Equations**

First order ordinary differential equations: Basic concepts and ideas, First order Exact differential equations, Integrating factors and rules to find integrating factors, Linear equations and Bernoulli equations, Initial value problems, Applications of first order differential equations: Orthogonal trajectories and Rate problems; Basic theory of higher order linear differential equations, Wronskian and its properties.

**Unit – 2 (12 hours)**

**Explicit Methods of Solving Higher-Order Linear Differential Equations**

Linear homogeneous equations with constant coefficients, Linear non-homogeneous equations, Method of undetermined coefficients, Method of variation of parameters, Two-point boundary value problems, Cauchy-Euler equations, System of linear differential equations.

### **Unit – 3**

**(18 hours)**

#### **First and Second Order Partial Differential Equations**

Classification and Construction of first-order partial differential equations, Method of characteristics and general solutions of first-order partial differential equations, Canonical forms and method of separation of variables for first order partial differential equations; Classification and reduction to canonical forms of second-order linear partial differential equations and their general solutions.

#### **Essential Readings**

1. Myint-U, Tyn and Debnath, Lokenath (2007). Linear Partial Differential Equations for Scientist and Engineers (4th ed.). Birkhäuser. Indian Reprint.
2. Ross, Shepley L. (1984). Differential Equations (3rd ed.). John Wiley & Sons.

#### **Suggestive Readings**

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson Education.
- Kreyszig, Erwin. (2011). Advanced Engineering Mathematics (10th ed.). Wiley India.
- Sneddon I. N. (2006). Elements of Partial Differential Equations. Dover Publications.

**Note:** Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

**B.A. (Prog.) Semester-IV with Mathematics as Major**  
**Category-II**

**DISCIPLINE SPECIFIC CORE COURSE (DSC-4): INTRODUCTION TO GRAPH THEORY**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Introduction to Graph Theory	4	3	1	0	Class XII pass with Mathematics	NIL

**Learning Objectives:** The primary objective of this course is to introduce:

- Problem-solving techniques using various concepts of graph theory.
- Various properties like planarity and chromaticity of graphs.
- Several applications of these concepts in solving practical problems.

**Learning Outcomes:** This course will enable the students to:

- Good familiarity with all initial notions of graph theory and related results and seeing them used for some real-life problems.
- Learning notion of trees and their enormous usefulness in various problems.
- Learning various algorithms and their applicability.
- Studying planar graphs, Euler theorem associated to such graphs and some useful applications like coloring of graphs.

**SYLLABUS OF DSC-4**

**UNIT-I: Graphs, Types of Graphs and Basic Properties (12 hours)**

Graphs and their representation, Pseudographs, Subgraphs, Degree sequence, Euler's theorem, Isomorphism of graphs, Paths and circuits, Connected graphs, Euler trails and circuits, Hamiltonian paths and cycles, Adjacency matrix, Weighted graphs, Travelling salesman problem, Dijkstra's algorithm.

**UNIT-II: Directed Graphs and Applications, Trees (18 hours)**

The Chinese postman problem; Digraphs, Bellman-Ford algorithm, Tournaments, Directed network, Scheduling problem; Trees and their properties, Spanning trees, Kruskal's algorithm, Prim's algorithm, Acyclic digraphs and Bellman's algorithm.

**UNIT-III: Planar Graphs, Graph Coloring and Network Flows (15 hours)**

Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring, Applications of graph

coloring, Circuit testing and facilities design, Flows and cuts, Max flow-min cut theorem, Matchings, Hall's theorem.

### Essential Reading

1. Goodaire, Edgar G., & Parmenter, Michael M. (2011). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.

### Suggestive Readings

- Bondy, J. A. & Murty, U.S.R. (2008), Graph Theory with Applications. Springer.
- Chartrand, Gary, & Zhang, P. (2012). A First Course in Graph Theory. Dover Publications.
- Diestel, R. (1997). Graph Theory (Graduate Texts in Mathematics). Springer Verlag.
- West, Douglas B. (2001). Introduction to graph theory (2nd ed.). Pearson India.

**DISCIPLINE SPECIFIC CORE COURSE – 4 (Discipline A-4): ABSTRACT ALGEBRA**

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Abstract Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

**Learning Objectives:** The primary objective of the course is to introduce:

- Modular arithmetic, fundamental theory of groups, rings, integral domains, and fields.
- Symmetry group of a plane figure, and basic concepts of cyclic groups.
- Cosets of a group and its properties, Lagrange's theorem, and quotient groups.

**Learning Outcomes:** This course will enable the students to:

- Appreciate ample types of groups present around us which explains our surrounding better, and classify them as abelian, cyclic and permutation groups.
- Explain the significance of the notion of cosets, normal subgroups and homomorphisms.
- Understand the fundamental concepts of rings, subrings, fields, ideals, and factor rings.

### SYLLABUS OF DISCIPLINE A-4

#### UNIT-I: Introduction to Groups (12 hours)

Modular arithmetic; Definition and examples of groups, Elementary properties of groups, Order of a group and order of an element of a group; Subgroups and its examples, Subgroup tests; Center of a group and centralizer of an element of a group.

#### UNIT-II: Cyclic Groups, Permutation Groups and Lagrange's Theorem (18 hours)

Cyclic groups and its properties, Generators of a cyclic group; Group of symmetries; Permutation groups, Cyclic decomposition of permutations and its properties, Even and odd



## B.Sc. (Prog.)/ BA (Prog.) Semester-IV with Mathematics as non-Major

### Category-III

#### DISCIPLINE SPECIFIC CORE COURSE – 4 (Discipline A-4): ABSTRACT ALGEBRA

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Abstract Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

**Learning Objectives:** The primary objective of the course is to introduce:

- Modular arithmetic, fundamental theory of groups, rings, integral domains, and fields.
- Symmetry group of a plane figure, and basic concepts of cyclic groups.
- Cosets of a group and its properties, Lagrange's theorem, and quotient groups.

**Learning Outcomes:** This course will enable the students to:

- Appreciate ample types of groups present around us which explains our surrounding better, and classify them as abelian, cyclic and permutation groups.
- Explain the significance of the notion of cosets, normal subgroups and homomorphisms.
- Understand the fundamental concepts of rings, subrings, fields, ideals, and factor rings.

#### SYLLABUS OF DISCIPLINE A-4

##### **UNIT-I: Introduction to Groups (12 hours)**

Modular arithmetic; Definition and examples of groups, Elementary properties of groups, Order of a group and order of an element of a group; Subgroups and its examples, Subgroup tests; Center of a group and centralizer of an element of a group.

##### **UNIT-II: Cyclic Groups, Permutation Groups and Lagrange's Theorem (18 hours)**

Cyclic groups and its properties, Generators of a cyclic group; Group of symmetries; Permutation groups, Cyclic decomposition of permutations and its properties, Even and odd permutations and the alternating group; Cosets and Lagrange's theorem; Definition and examples of normal subgroups, Quotient groups; Group homomorphisms and properties.

##### **UNIT-III: Rings, Integral Domains and Fields (15 hours)**

Definition, examples and properties of rings, subrings, integral domains, fields, ideals and factor rings; Characteristic of a ring; Ring homomorphisms and properties.

### Essential Reading

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).

### Suggestive Reading

- Beachy, John A., & Blair, William D. (2006). Abstract Algebra (3rd ed.). Waveland Press.

## B.Sc. (Physical Sciences/Mathematical Sciences) Semester-IV with Mathematics as one of the Core Discipline Category-III

### DISCIPLINE SPECIFIC CORE COURSE – 4 (Discipline A-4): ABSTRACT ALGEBRA

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Abstract Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

**Learning Objectives:** The primary objective of the course is to introduce:

- Modular arithmetic, fundamental theory of groups, rings, integral domains, and fields.
- Symmetry group of a plane figure, and basic concepts of cyclic groups.
- Cosets of a group and its properties, Lagrange's theorem, and quotient groups.

**Learning Outcomes:** This course will enable the students to:

- Appreciate ample types of groups present around us which explains our surrounding better, and classify them as abelian, cyclic and permutation groups.
- Explain the significance of the notion of cosets, normal subgroups and homomorphisms.
- Understand the fundamental concepts of rings, subrings, fields, ideals, and factor rings.

#### SYLLABUS OF DISCIPLINE A-4

##### **UNIT-I: Introduction to Groups (12 hours)**

Modular arithmetic; Definition and examples of groups, Elementary properties of groups, Order of a group and order of an element of a group; Subgroups and its examples, Subgroup tests; Center of a group and centralizer of an element of a group.

##### **UNIT-II: Cyclic Groups, Permutation Groups and Lagrange's Theorem (18 hours)**

Cyclic groups and its properties, Generators of a cyclic group; Group of symmetries; Permutation groups, Cyclic decomposition of permutations and its properties, Even and odd

## B.A. (Prog.) Semester-V with Mathematics as Major

### Category-II

#### DISCIPLINE SPECIFIC CORE COURSE (DSC-5): LINEAR PROGRAMMING

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Programming	4	3	1	0	Class XII pass with Mathematics	NIL

**Learning Objectives:** The primary objective of this course is to introduce:

- The solution of linear programming problem using simplex method.
- The solution of transportation and assignment problems.
- Game theory which makes possible the analysis of the decision-making process of two interdependent subjects.

**Learning Outcomes:** This course will enable the students to:

- Learn about the simplex method used to find optimal solutions of linear optimization problems subject to certain constraints.
- Write the dual of a linear programming problem.
- Solve the transportation and assignment problems.
- Learn about solution of rectangular games using graphical method and dominance.
- Formulate game to a pair of associated prima-dual linear programming problems.

#### **SYLLABUS OF DSC-5**

#### **UNIT-I: Linear Programming Problem, Simplex Method, and Duality (18 hours)**

Standard form of the LPP, graphical method of solution, basic feasible solutions, and convexity; Introduction to the simplex method: Optimality criterion and unboundedness, Simplex tableau and examples, Artificial variables; Introduction to duality, Formulation of the dual problem with examples.

#### **UNIT-II: Transportation and Assignment Problems (15 hours)**

Definition of transportation problem, finding initial basic feasible solution using Northwest-corner method, Least-cost method, and Vogel approximation method; Algorithm for solving transportation problem; Hungarian method of solving assignment problem.

#### **UNIT-III: Two-Person Zero-Sum Games (12 hours)**

Introduction to game theory, rectangular games, Mixed strategies, Dominance principle; Formulation of game to primal and dual linear programming problems.

#### **Essential Readings**

3. Thie, Paul R., & Keough, G. E. (2014). An Introduction to Linear Programming and Game Theory. (3rd ed.). Wiley India Pvt. Ltd.
4. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

### Suggestive Readings

- Hadley, G. (1997). Linear Programming. Narosa Publishing House. New Delhi.
- Hillier, F. S., & Lieberman, G. J. (2021). Introduction to Operations Research (11th ed.). McGraw-Hill Education (India) Pvt. Ltd.

## DISCIPLINE SPECIFIC CORE COURSE – 5 (Discipline A-5): ELEMENTS OF REAL ANALYSIS

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Real Analysis	4	3	1	0	Class XII pass with Mathematics	NIL

**Learning Objectives:** The primary objective of this course is to introduce:

- The real line with algebraic, order and completeness properties.
- Convergence and divergence of sequences and series of real numbers with applications.

**Learning Outcomes:** This course will enable the students to:

- Understand the basic properties of the set of real numbers, including completeness and Archimedean with some consequences.
- Recognize bounded, convergent, monotonic and Cauchy sequences
- Learn to apply various tests such as limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

### SYLLABUS OF DISCIPLINE A-5

#### UNIT-I: Basic Properties of the Set of Real Numbers (12 hours)

Field and order properties of  $\mathbb{R}$ , basic properties and inequalities of the absolute value of a real number, bounded above and bounded below sets, Suprema and infima, The completeness axiom and the Archimedean property of  $\mathbb{R}$ .

#### UNIT-II: Real Sequences (18 hours)

Convergence of a real sequence, Algebra of limits, The squeeze principle and applications, Monotone sequences, Monotone convergence theorem and applications, Cauchy sequences, Cauchy criterion for convergence and applications.

**Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.**

## **B.Sc. (Prog.)/ BA (Prog.) Semester-V with Mathematics as non-Major**

### **Category-III**

#### **DISCIPLINE SPECIFIC CORE COURSE – 5 (Discipline A-5): ELEMENTS OF REAL ANALYSIS**

#### **CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Real Analysis	4	3	1	0	Class XII pass with Mathematics	NIL

**Learning Objectives:** The primary objective of this course is to introduce:

- The real line with algebraic, order and completeness properties.
- Convergence and divergence of sequences and series of real numbers with applications.

**Learning Outcomes:** This course will enable the students to:

- Understand the basic properties of the set of real numbers, including completeness and Archimedean with some consequences.
- Recognize bounded, convergent, monotonic and Cauchy sequences
- Learn to apply various tests such as limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

#### **SYLLABUS OF DISCIPLINE A-5**

##### **UNIT-I: Basic Properties of the Set of Real Numbers (12 hours)**

Field and order properties of  $\mathbb{R}$ , basic properties and inequalities of the absolute value of a real number, bounded above and bounded below sets, Suprema and infima, The completeness axiom and the Archimedean property of  $\mathbb{R}$ .

##### **UNIT-II: Real Sequences (18 hours)**

Convergence of a real sequence, Algebra of limits, The squeeze principle and applications, Monotone sequences, Monotone convergence theorem and applications, Cauchy sequences, Cauchy criterion for convergence and applications.

##### **UNIT-III: Infinite Series of Real Numbers (15 hours)**

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence of series, Tests for convergence of positive term series, Applications of the integral test, Comparison tests, D'Alembert's ratio test, Cauchy's  $n$ th root test, Raabe's test; Alternating series, Leibniz alternating series test, Absolute and conditional convergence.

### **Essential Reading**

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

### **Suggestive Readings**

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

**B.A. (Prog.) Semester-VI with Mathematics as Major**  
**Category-II**

**DISCIPLINE SPECIFIC CORE COURSE (DSC-6): ELEMENTARY MATHEMATICAL ANALYSIS**

**CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE**

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elementary Mathematical Analysis	4	3	1	0	Class XII pass with Mathematics	Discipline A-5: Elements of Real Analysis

**Learning Objectives:** The primary objective of this course is to introduce:

- Sequential criterion for limits and continuity of real-valued functions.
- Riemann integral of real-valued function  $f$  on  $[a, b]$  using Darboux sums.
- Pointwise and uniform convergence of sequences and series of functions.

**Learning Outcomes:** This course will enable the students to:

- Apply sequential continuity criterion for the proof of intermediate value theorem.
- Understand the basic tool used to calculate integrals.
- Apply uniform convergence for term-by-term integration in power series expansion.

**SYLLABUS OF DSC-6**

**UNIT-I: Continuous Functions (12 hours)**

Sequential criterion for limits and continuity of functions, Continuity on intervals, Intermediate value theorem and applications; Uniform continuity.

**UNIT-II: The Riemann Integral (15 hours)**

Riemann integration, criterion for integrability and examples; Integrability of continuous and monotone functions, Algebraic properties of the Riemann integral, Fundamental theorem of calculus (first form).

**UNIT-III: Uniform Convergence (18 hours)**

Sequences and series of functions: Pointwise and uniform convergence, Uniform Cauchy criterion, Weierstrass M-test, Implications of uniform convergence in calculus; Power series, Radius and interval of convergence, Applications of Abel's theorem for power series.

## Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

## Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

## DISCIPLINE SPECIFIC CORE COURSE – 6 (Discipline A-6): PROBABILITY AND STATISTICS

### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Probability and Statistics	4	3	0	1	Class XII pass with Mathematics	NIL

**Learning Objectives:** The primary objective of this course is to:

- Make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness.
- Render the students to several examples and exercises that blend their everyday experiences with their scientific interests to form the basis of data science.

**Learning Outcomes:** This course will enable the students to:

- Understand some basic concepts and terminology-population, sample, descriptive and inferential statistics including stem-and-leaf plots, dotplots, histograms and boxplots.
- Learn about probability density functions and various univariate distributions such as binomial, hypergeometric, negative binomial, Poisson, normal, exponential, and lognormal.
- Understand the remarkable fact that the empirical frequencies of so many natural populations, exhibit bell-shaped (i.e., normal) curves, using the Central Limit Theorem.
- Measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.

## SYLLABUS OF DISCIPLINE A-6

### UNIT-I: Descriptive Statistics, Probability, and Discrete Probability Distributions (15 hours)

Descriptive statistics: Populations, Samples, Stem-and-leaf displays, Dotplots, Histograms, Qualitative data, Measures of location, Measures of variability, Boxplots; Sample spaces



## B.Sc. (Prog.)/ BA (Prog.) Semester-VI with Mathematics as non-Major

### Category-III

#### DISCIPLINE SPECIFIC CORE COURSE-6 (Discipline A-6): PROBABILITY AND STATISTICS

#### CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Probability and Statistics	4	3	0	1	Class XII pass with Mathematics	NIL

**Learning Objectives:** The primary objective of this course is to:

- Make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness.
- Render the students to several examples and exercises that blend their everyday experiences with their scientific interests to form the basis of data science.

**Learning Outcomes:** This course will enable the students to:

- Understand some basic concepts and terminology-population, sample, descriptive and inferential statistics including stem-and-leaf plots, dotplots, histograms and boxplots.

- Learn about probability density functions and various univariate distributions such as binomial, hypergeometric, negative binomial, Poisson, normal, exponential, and lognormal.
- Understand the remarkable fact that the empirical frequencies of so many natural populations, exhibit bell-shaped (i.e., normal) curves, using the Central Limit Theorem.
- Measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.

## **SYLLABUS OF DISCIPLINE A-6**

### **UNIT-I: Descriptive Statistics, Probability, and Discrete Probability Distributions (15 hours)**

Descriptive statistics: Populations, Samples, Stem-and-leaf displays, Dotplots, Histograms, Qualitative data, Measures of location, Measures of variability, Boxplots; Sample spaces and events, Probability axioms and properties, Conditional probability, Bayes' theorem, and independent events; Discrete random variables & probability distributions, Expected values; Probability distributions: Binomial, geometric, hypergeometric, negative binomial, Poisson, and Poisson distribution as a limit.

### **UNIT-II: Continuous Probability Distributions (15 hours)**

Continuous random variables, Probability density functions, Uniform distribution, Cumulative distribution functions and expected values, The normal, exponential, and lognormal distributions.

### **UNIT-III: Central Limit Theorem and Regression Analysis (15 hours)**

Sampling distribution and standard error of the sample mean, Central Limit Theorem, and applications; Scatterplot of bivariate data, Regression line using principle of least squares, Estimation using the regression lines; Sample correlation coefficient and properties.

### **Practical (30 hours)**

Software labs using Microsoft Excel or any other spreadsheet.

- 1) Presentation and analysis of data (univariate and bivariate) by frequency tables, descriptive statistics, stem-and-leaf plots, dotplots, histograms, boxplots, comparative boxplots, and probability plots ([1] Section 4.6).
- 2) Fitting of binomial, Poisson, and normal distributions.
  - 3) Illustrating the Central Limit Theorem through Excel.
  - 4) Fitting of regression line using the principle of least squares.
  - 5) Computation of sample correlation coefficient.

### **Essential Reading**

1. Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences (9th ed.). Cengage Learning India Private Limited. Delhi. Indian Reprint 2022.